NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1320

INVESTIGATIONS ON WINGS WITH AND WITHOUT SWEEPBACK AT HIGH SUBSONIC SPEEDS

By Jakob Ackeret, Max Degen, and Nikolaus Rott

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INVESTIGATIONS ON WINGS WITH AND WITHOUT SWEEPBACK AT HIGH SUBSONIC SPEEDS*

By Jakob Ackeret, Max Degen, and Nikolaus Rott[1]

SUMMARY

In the high-speed wind tunnel of the Institute for Aerodynamics, E.T.H., Zürich, straight and sweptback wings have been investigated at high subsonic speeds. Drag measurements at zero incidence were made on a series of geometrically similar models at the same Reynolds number which was maintained constant by change of density. By this, theoretical tunnel correction formulas could be checked and an extrapolation to vanishing tunnel influence was possible; straight and sweptback wings were compared after corrections.

Two different profile thicknesses (9 and 12 percent) have been investigated. The transonic drag Mach number relation for different thicknesses was found to be in a very satisfactory agreement with von Kármán's similarity law.

INTRODUCTION

The experimental investigation of objects in the wind tunnel is known to offer special difficulties in case of speeds approaching sonic velocity. The basic reason lies in the fact that in this velocity range an extraordinary sensitivity of the flow to small cross-sectional changes exists. For a closed tunnel, the continuity equation

$$fU\rho = \text{constant}$$

together with the formula of sonic velocity

$$a^2 = \frac{dp}{d\rho}$$

*"Untersuchungen an gepfeilten und ungepfeilten Flügeln bei hohen Unterschallgeschwindigkeiten." Zeitschrift für angewandte Mathematik und Physik (ZAMP) vol. 1, 1950, pp. 32-42. (Verlag Birkhäuser, Basel, Switzerland, publisher and copyright holder.)

[1]Zürich, Institut für Aerodynamik, ETH.

and the equation of motion

$$U\,dU = -\frac{dp}{\rho}$$

yields the simple relation

$$\frac{dU}{U} = -\frac{df}{f}\,\frac{1}{1 - M^2}$$

where $M = U/a$ denotes the Mach number.

For $M \longrightarrow 1$ the sensitivity therefore becomes infinitely large. Consequently, in a tunnel of constant cross section, sonic velocity cannot be attained ahead of the model; it appears in the region of the largest cross section of the model; the tunnel is then blocked. Higher free-stream Mach numbers are not possible for steady flow.

This boundary case shows that the finite magnitude of the tunnel will lead to differences compared to the flow condition in the free atmosphere. Much effort has been put forth to make the transition, by suitable numerical corrections, from the test values in the tunnel of finite magnitude to the case undisturbed by walls.

It is true that there exists, on principle, the possibility of forming the tunnel walls in such a manner that they fit the desired flow. If we knew what the stream lines for the desired Mach number look like, the solidification of any stream surfaces outside of the model would not produce a change in the flow (except for friction effects at the tunnel walls). A measurement free from disturbances would result; however, the fact that the flow pattern at first is known only inaccurately and that it depends on the Mach number and on the position of the model with respect to the free-stream direction makes the "trial and error" method it necessitates extremely laborious and time consuming.

Another procedure is described by Feldmann[2]. He investigates three geometrically similar models of different size for the same Reynolds numbers and extrapolates to the model size of zero or the tunnel width of infinity. Since with decreasing magnitude of the model the tunnel disturbances tend toward zero, the extrapolation probably is fundamentally permissible; however, one cannot use arbitrarily small models, and there arises the question of what the course of the extrapolation will be in case of vanishing model size.

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[2]F. K. Feldmann, Untersuchung von symmetrischen Tragflügelprofilen bei hohen Unterschallgeschwindigkeiten in einem geschlossenen Windkanal (Investigation of symmetrical wing profiles at high subsonic speeds in a closed wind tunnel). Mitteilungen Institut Aerodynamik ETH, No. 14, Leemann, Zürich 1948.

In the discussion, this problem will be treated first. A secondary problem of some significance should be mentioned. As is well known, the critical phenomena on wings are shifted to higher Mach numbers if the wing is swept back. This can be immediately confirmed in a wind-tunnel test; however, one could be led to assume that the sweepback effect measured is at least partly influenced by the tunnel. Here it is of advantage to make a direct comparison between the values, extrapolated according to Feldmann, of unswept and swept wings (for the same profiles and Reynolds numbers). Likewise, it will be possible to examine the so-called Kármán rule if profiles of different thickness are investigated.

1. THE CALCULATION METHODS OF TUNNEL CORRECTION

Any tunnel correction for zero lift, which is the case always considered below, is based on the assumption that the flow in the closest proximity of the model is the same as in the free atmosphere, that, however, the free-stream velocity at some distance ahead of the object in the tunnel is different from that in free atmosphere. Thus, one must determine what difference exists between the mean local speed in the proximity of the model location and the tunnel speed ahead of the model. This can be done by first determining, in the well-known manner, the influence of the fixed walls by doubly periodic mirroring and then, with the methods of linearized theory, taking into consideration the compressibility. If the flow were free from drag, the model could be represented by a suitable source-sink distribution, the mirror images of which may be assumed to be simple dipoles since the model always is relatively small compared to the tunnel dimensions. The equivalent dipole strength is proportional to the volume of the object. The incremental velocity ΔU_{i_1} may be calculated simply; the result is

$$\frac{\Delta U_{i_1}}{U_o} = \epsilon_{i_1} = k \frac{V}{F_K^{3/2}} \tag{1}$$

wherein

V signifies the model volume

F_K the tunnel cross section

k a numerical factor

U_o the uncorrected measured free-stream velocity in the tunnel

The numerical factor k is a function of the form of the test cross section and of the model. For smaller models, however, the model shape is unimportant, as mentioned above. Thus, Thom[3] finds for bodies of revolution and wings of finite span and medium fineness the value $k = 0.90$ for a rectangular tunnel with the height-width ratio = 0.7. For quadratic tunnels, a check calculation according to the same method resulted in a value $k = 0.81$.

If the model shows a drag, there forms behind it a layer of reduced speed, the profile of which is denoted as "depression." To some approximation, as is necessary for corrections, the displacement effect of the wake may be replaced by that of a simple source flow. (After all, one is here concerned only with the effect of the mirrored sources and for this the details of the displacement procedure are unessential.) The free-stream velocity in the tunnel far ahead of the model is thereby reduced by the amount ΔU_{i_2} compared to the free atmosphere. One obtains

$$\frac{\Delta U_{i_2}}{U_o} = \epsilon_{i_2} = \frac{c_w}{4} \frac{F_B}{F_K} \tag{2}$$

where F_B is the surface of reference (for instance, wing-plan area) and

$$c_w = \frac{W_{meas}}{F_B \rho U_o^2/2}$$

Between the measured free-stream velocity U_o in the tunnel and the equivalent free-stream velocity in the unlimited stream there exists, therefore, (calculated for incompressible conditions) a difference

$$\frac{\Delta U_i}{U_o} = \epsilon_{i_1} + \epsilon_{i_2} \tag{3}$$

In order to take the compressibility into consideration, the conversion may be carried out in linearized approximation according to the generalized Prandtl rule. According to Göthert[4] the result is

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[3] A. Thom, Blockage Corrections in a Closed High-Speed Tunnel. R. & M. No. 2033, 1943.

[4] B. Göthert, Windkanal-Korrekturen bei hohen Luftgeschwindigkeiten (Wind-tunnel corrections at high airspeeds). Lilenthal-Gesellschaft für Luftfahrtforschung Bericht 127, 1940, p. 114.

$$\frac{\Delta U_K}{U_o} = \epsilon_K = \frac{\epsilon_{i_1}}{\left(1 - M_o^2\right)^{3/2}} + \frac{\epsilon_{i_2}}{1 - M_o^2} \tag{4}$$

M_o being the Mach number corresponding to U_o.

Thom[5] gives the expression

$$\frac{\Delta U_K}{U_o} = \epsilon_K = \frac{\epsilon_{i_1} + \epsilon_{i_2}}{\left(1 - M_o^2\right)^{3/2}} \tag{5}$$

without exact substantiation. If the replacement of the wake by a simple source is taken as reliable, the conversion formula of Göthert appears theoretically better substantiated.

For the correction of the Mach number one obtains

$$\frac{\Delta M}{M_o} = \epsilon_K \left(1 + \frac{k - 1}{2} M_o^2\right) \tag{6}$$

from the adiabatic connection between U and M (for small ΔU and ΔM).

Finally, the drag coefficient c_w may be converted to the value corresponding to the dynamic pressure $q = \frac{1}{2}\rho U^2$ pertaining to the corrected value of M; there U as well as ρ is corrected correspondingly. If one would refer the drag force to the stagnation pressure p_K, thus form

$$c_{w_K} = \frac{W_{meas}}{p_k F_B}$$

c_{w_K} would be independent of the ratio of tunnel dimensions to model dimensions if the corrected Mach number is the same; therefore,

$$c_{w_{corr}} = c_{w_{meas}} \frac{q_{meas}}{q_{corr}}$$

[5]See footnote 3 on page 4.

The variation of the drag coefficient c_W against the Mach number M for different model sizes but constant Reynolds number is assumed to be known. According to our basic assumption, there are on the c_w curves of two different models corresponding points at which the flow in the closest proximity of the model is similar for different free-stream Mach numbers. For these two points the values

$$c_{w_{meas}} \frac{q_{meas}}{p_K}$$

should therefore be of the same magnitude. The hypothesis may be tested by determining and comparing the flow state at both points. This can be done to a certain degree (as already mentioned by Feldmann[6]) by a comparison of the schlieren pictures of both states. Figure 1 represents such corresponding pictures copied on the same wing chord for two models which show rather good agreement. Although the differences in the Mach numbers seem insignificant, the highly similar appearance of the schlieren photographs represents a sensitive criterion. Figure 2 shows that the schlieren picture of a model changes noticeably even for small Mach variations.

2. MEASUREMENTS

First, measurements were made, in continuation of Feldmann's report (footnote 2), on wings of rectangular plan form, aspect ratio 3.25, with profiles of 12-percent thickness NACA 0-0012-1.1-30 or of 9-percent thickness NACA 0-0009-1.1-30. In addition, wings with 35° sweepback were investigated which had the same profiles in flight direction as the corresponding rectangular wings as well as the same constant chord and span. The measurements were made on similar models of different size.

Wing number	Span in millimeters	Chord in millimeters
1	260	80
2	211	65
3	162.5	50

Measuring series

AI:	Unswept wing	Profile 12 percent	Wing number 1, 2, 3
AII:	Unswept wing	Profile 9 percent	Wing number 1, 2
BI:	Wing with 35° sweepback	Profile 12 percent	Wing number 1, 2, 3
BII:	Wing with 35° sweepback	Profile 9 percent	Wing number 1, 2, 3

[6]See footnote 2 on page 2.

An accurate description of the test arrangements (test section, supports, balance) may be found in Feldmann's report (footnote 2). Here we shall only describe a few details.

(a) Test Section

In the test section with built-in supports, without model, in flow direction, constant static pressure is supposed to prevail. In order to obtain this and to eliminate as far as possible any blockage effects and effects opposed to blockage, the displacement volume of the two sweptback supports was compensated and the cross-sectional area for the air passage kept constant. This is achieved by cutting out at four wooden corner strips placed in the quadratic test section ($F_K = 1560\ cm^2$). The maximum cut-out area was 8.6 centimeters^2; it corresponded to the maximum displacement cross section of the supports.

The free-stream Mach number was fixed by measuring the stagnation pressure p_K in the box (tank) and determining the pressure difference between tank and test section. The static pressure connection in the test section was, according to the model size, provided 230 to 245 millimeters ahead of the leading edge of the model at the upper side of the tunnel. As measuring instruments, mercury manometers were used, the reading accuracy of which was 1/20 mm Hg. The regulation to constant Reynolds number was accomplished by variation of the density ($Re \cong 400,000$).

(b) Calibration of the Supports

Since the drags of the supports are percentually high compared to the profile drags, an accurate calibration is necessary. The effect of the wing on the supports and vice versa has to be taken into consideration so that the calibrations of the supports should be performed under conditions as nearly similar as possible to those occurring later for the measurements. Hence, models which had been made of wood for that purpose were mounted rigidly in the tunnel over the supports on two thin auxiliary struts. Small cut-outs at the lower side of the wing allowed the supports to oscillate to and fro with a play of 2 millimeters, and to shield the screw threads, which protruded into the wing, from the air stream.

Since, by the installation of the auxiliary struts, the tunnel cross section is reduced and thereby the blockage Mach number changed, one must, in calibrating the supports, be particularly careful to keep the cross-sectional area of the test section constant by adequate removal of material at the upper corner strips.

Two different supports were used which in the parts adjoining the wing were of different chord so that the geometrical similarity of the wing connection was approximately guaranteed for the two larger models.

3. DISCUSSION OF THE MEASURING RESULTS

The same corrected Mach number should pertain to points with the same $c_w q/p_K$ measured on models of different sizes and hence at different free-stream Mach numbers in the tunnel. By fulfillment of this condition, the numerical corrections can thus be obtained experimentally.

The Mach numbers measured for different model sizes for $c_w q/p_K$ = const. were plotted against the ratio of frontal area to tunnel area ($= F_s/F_K$) of the models (figs. 3 and 4, upper half, open circles connected by solid lines). The calculated corrected Mach numbers in each case (filled-in circles - according to Thom, equation (5), filled-in triangles - according to Göthert, equation (4)) also were plotted against F_s/F_K. They should be the same for all models, that is, their connecting lines (thinly drawn, Thom, dashed, Göthert) ought to stand vertically in figures 3 and 4. One can see that, according to this criterion, the order of magnitude of the calculated corrections appears in general to be correct; only a few of the curves show a slight inclination. With decreasing model size we may attach increasing significance to the corrected Mach numbers. If, therefore, the thin connecting line of the M_{corr} with decreasing F_s/F_K is inclined toward higher Mach numbers, the correction is too small and vice versa. One realizes that the correction according to Thom is somewhat too small up to about $M_{corr} = 0.84$, too large above $M_{corr} = 0.93$. According to our measurements, this results independently of thickness and wing shape. Göthert's corrections, which differ noticeably from Thom's only for higher values of c_w, are there too small up to about $M_{corr} = 0.93$.

In order to obtain a result also in those cases where the calculated corrections do not lead to the same value, an extrapolation according to Feldmann is carried out. The measured Mach numbers against F_s/F_K are connected by a curve (heavily drawn lines in figs. 3 and 4, upper part) and extrapolated to $F_s/F_K = 0$. Therein, the tendency of the theoretical correction is taken into account concerning the limiting behavior toward $F_s/F_K = 0$. It is then shown that for Mach numbers up to about 0.84, the extrapolation curves slightly curve toward small M and that they may be drawn practically rectilinearly beyond that value. Above $M_{corr} = 0.92$ (which occurred in our measurements only in case of sweptback wings), the extrapolation curves were supposed, according to the correction calculated according to Thom, to start curving toward higher M; however, in the present report, we extrapolated as rectilinearly as possible up to the highest Mach numbers. The curvature required by the calculated correction is caused by the very rapid increase of the factor $(1 - M^2)^{-3/2}$ in the proximity of $M = 1$. For decreasing F_s/F_K the Mach number to be inserted in the corrections (4) and (5) increases more and more; however, it is hardly possible to take, so near to $M = 1$, a factor fully into consideration which is obtained according to the linearized theory and for $M \longrightarrow 1$ leads to infinity. Kármán's rule supported our procedure. (Compare below.)

For these highest Mach numbers, the measurements lie almost at the blockage limit; however, if one follows the measured Mach numbers for $c_w q/p_K$ = constant up over F_s/F_K one can see them receding farther and farther from the blockage limit with decreasing F_s/F_K. Thus one may assume that in this region the values become more reliable precisely by the extrapolation.

In figures 3 and 4, lower part, the extrapolated curves c_w over M are heavily drawn. The comparison of unswept and sweptback wings shows that the difference in the Mach numbers for equal c_w is somewhat larger still for the extrapolated than for the measured curves. The swept-wing measurement of transonic drag increase thus is more strongly affected by the tunnel although the blockage is decreasing since the increase occurs at higher Mach numbers and the tunnel corrections are therefore larger. If Thom's correction were used up to the highest Mach numbers (figs. 3 and 4, dash-dotted), the difference would be still larger.

4. SIMILARITY THEORY OF TRANSONIC FLOW

According to Kármán's similarity theory of transonic flow[7], it should be possible to make the curves c_w over M for profiles that are similar but of different thickness coincide by means of a change in scale. For plane flow, the following rule applies: If the ratio of profile thickness d and chord t is denoted by $d/t = \tau$, the curves $c_w/\tau^{5/3}$ over $(1 - M)/\tau^{2/3}$ should coincide for various thicknesses. Only that part of c_w which corresponds to the pressure drag in transonic flow (combined with weak shocks) is to be considered. Hence, the constant part c_{w_0} of the friction drag for smaller Mach numbers was subtracted throughout. For $\tau = 0.12$, $c_{w_0} = 0.008$, for $\tau = 0.09$, $c_{w_0} = 0.0072$.

Application of Kármán's rule to the extrapolated values showed very satisfactory results for the unswept as well as for the sweptback wing (fig. 5). For the sweptback wing, the rule was applied also to the variations corrected according to Thom where a less satisfactory agreement resulted. This may be mentioned in support of the extrapolation

[7]Th. von Kármán, The Similarity Law of Transonic Flow. J. Math. Phys. 26, 1947, pp. 182-190.

used. It is remarkable that Kármán's rule seems to be valid also for the finite aspect ratios and shock strengths we dealt with in this report.

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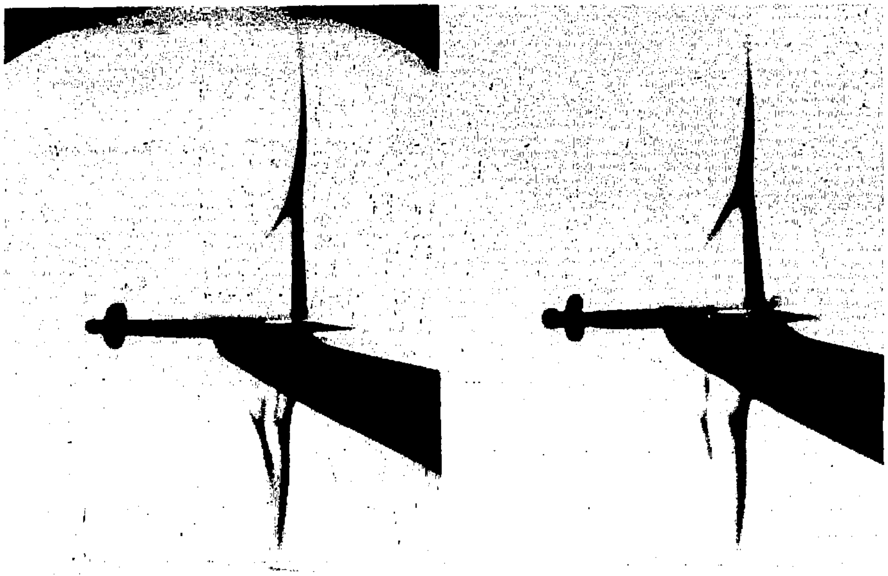

Figure 1.- Schlieren photographs on two geometrically similar unswept wings with a 9-percent profile thickness. (a) Model size No. 1; (b) Model size No. 2, copied on the same chord. The Mach number measured in the tunnel is: (a) $M_{meas} = 0.89$, (b) $M_{meas} = 0.90$; the corrected Mach number is in both cases $M_{corr} = 0.919$. The schlieren pictures are, to a great extent, identical.

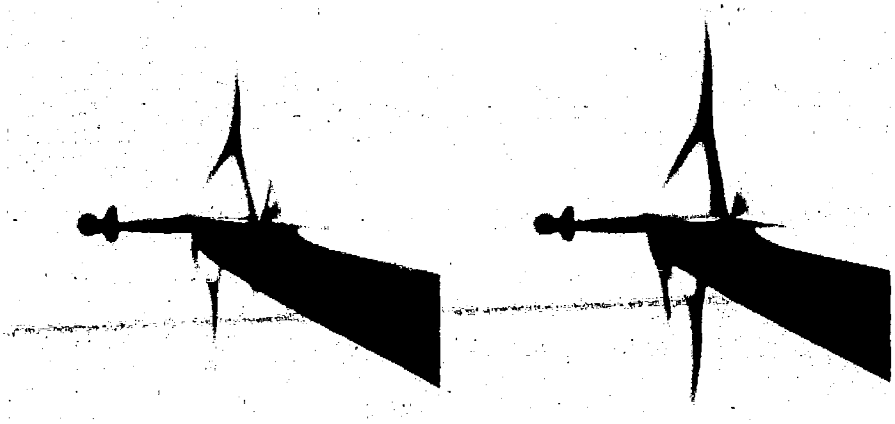

Figure 2.- Schlieren photographs on the unswept wing with 9-percent profile thickness, model size No. 2 for different Mach numbers (a) $M_{corr} = 0.885$; (b) $M_{corr} = 0.905$.

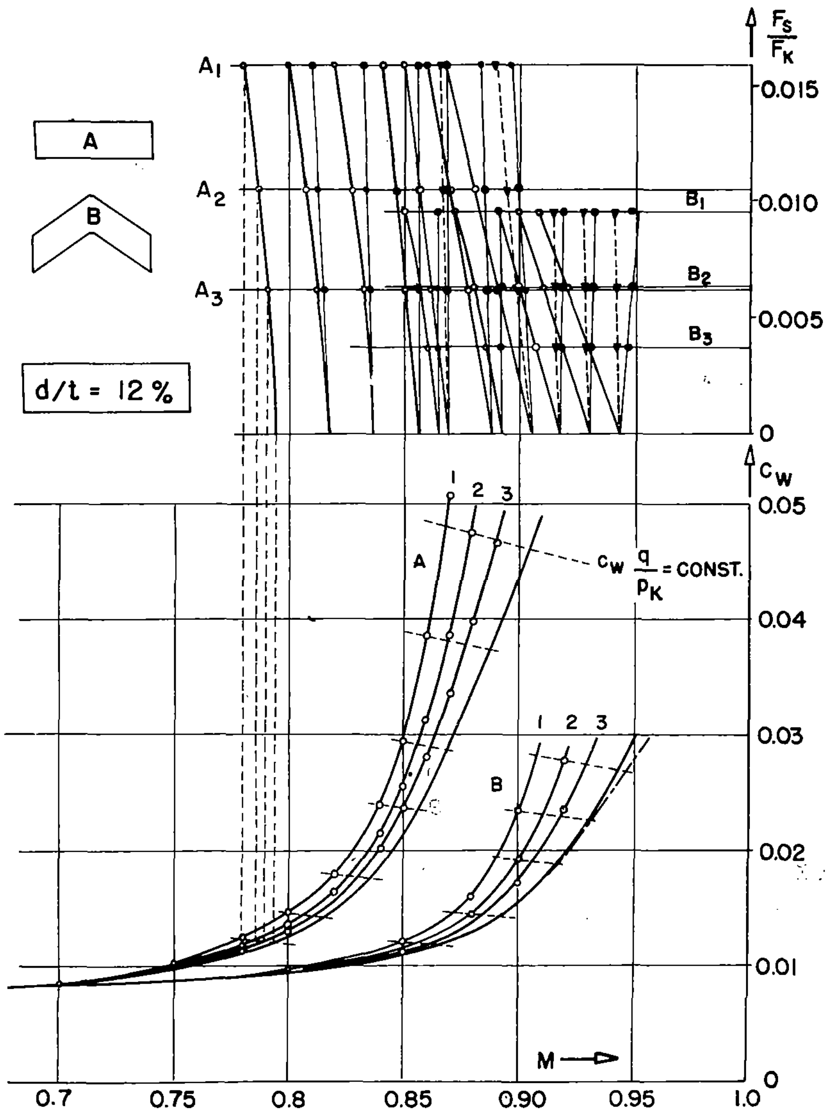

Figure 3.- Drag coefficient c_w plotted against the Mach number M for profile of 12-percent thickness. A. Unswept wing. B. Wing with 35° sweepback. Measuring series on wings of different model sizes, Nos. 1, 2, and 3. Solidly drawn: extrapolated curve c_w plotted against M. Dash-dotted: corrected according to Thom. Upper half: relation between Mach number and (frontal area:tunnel area =) F_S/F_K for $c_w q/p_K$ = constant, represented by open circles, connected by heavily drawn extrapolation lines; corrected Mach numbers according to Thom (filled-in circles, connected by thinly drawn lines) and according to Göthert (filled-in triangles connected by thin dashed lines).

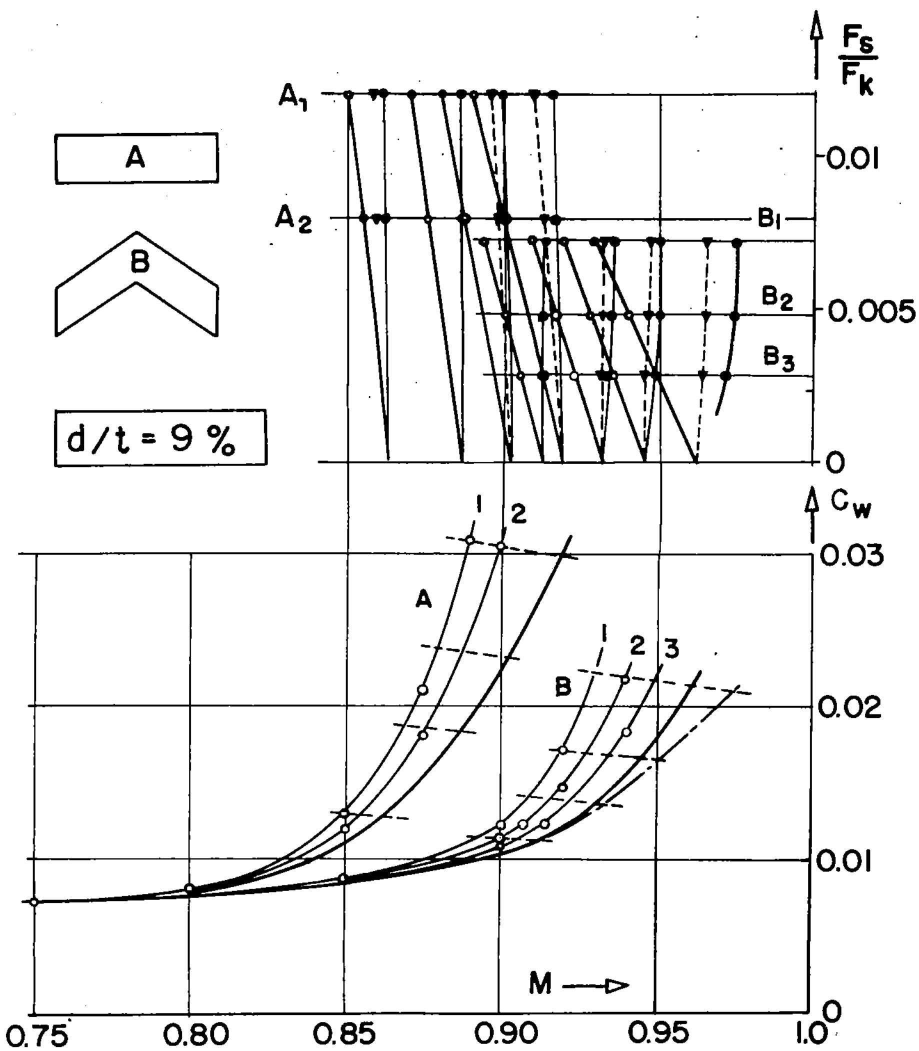

Figure 4.- Drag coefficient c_w plotted against the Mach number M for profile of 9-percent thickness. A. Unswept wing. B. Wing with 35° sweepback. Measuring series on wings of different model sizes, Nos. 1, 2, and 3. Solidly drawn: extrapolated curve c_w plotted against M. Dash-dotted: corrected according to Thom. Upper half: relation between Mach number and (frontal area:tunnel area =) F_S/F_K for $c_w q/p_K$ = constant, represented by open circles, connected by heavily drawn extrapolation lines; corrected Mach numbers according to Thom (filled-in circles, connected by thinly drawn lines) and according to Göthert (filled-in triangles connected by thin dashed lines).

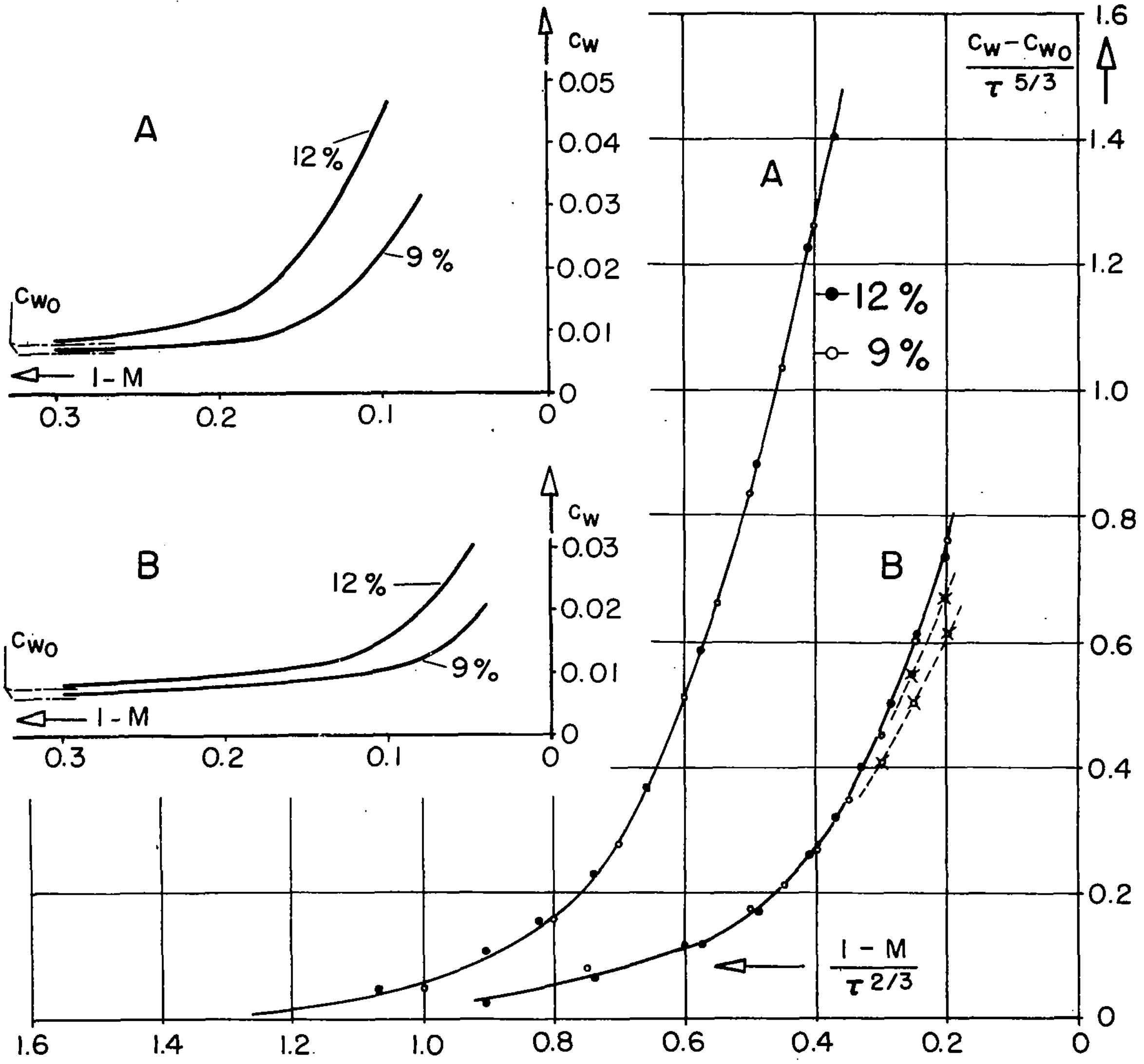

Figure 5.- Kármán's rule. A. Unswept wing. B. Wing with 35° sweepback. Filled-in circles: $\tau = 0.12$; open circles: $\tau = 0.09$. The same applies to crossed circles which pertain to the values for the sweptback wing corrected according to Thom. For comparison the extrapolated curves also have been drawn over the undisturbed scales.